including, with little additional mathematical complication, effects such as wall roughness, surface heating/cooling, and compressibility.

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New Form for an Adaptive Observer

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Introduction

THE problem considered here is the estimation of the states and parameters of an nth order linear time-invariant plant where only the input and output can be observed. This development is an extension of the results of Lion[1] and Luders.[3] Lion considered parameter identification without state estimation. Luders[3] related the "observer"[2] state estimations structure to Lion's algorithm resulting in a state and parameter estimation algorithm. The adaptive observer formulation presented here has the advantage of greater "state variable filter"[1] separation than given in Ref. 3. In addition this formulation has greater design freedom in "state variable filter" selection.

Brief Development

It is assumed that the completely observable system can be described by an nth-order time-invariant vector differential equation. For the sake of simplicity, the new canonical form is derived only for the single-input single-output case. Nevertheless the extension of this canonical form to the multi-input case is straightforward.[4]

Given

A stable stationary observable system transfer function with unknown parameters

$$G(s) = \sum_1^n \beta_i s^{i-1} \Big/ \left(s^n + \sum_1^n \alpha_i s^{i-1} \right) \tag{1}$$

find a convergent parameter and state estimator.

Solution

Restriction of Lion's[1] "state variable filter" to a simple form leads to a state estimate (observer) relationship. The transfer function Eq. (1) can be expressed more conveniently in terms of *known parameters* (λ) as follows:

$$G(s) = \frac{\left(b_n + \sum_1^{n-1} M_i b_i \right)}{\left(s + a_n + \lambda_n + \sum_1^{n-1} M_i a_i \right)} \tag{2}$$

where

$$M_i \triangleq \prod_{j=i}^{n-1} \frac{1}{(s+\lambda_j)}$$

In expression (2) the (a, b) are now the parameters to be identified. The transformation relating (a, b) to (α, β) involving (λ) can be derived easily by equating coefficients of like powers of s. Note that if all $\lambda_i = 0$ then $(a, b) = (\alpha, \beta)$. The form of Eq. (2) is motivated by state estimate convergence requirement.

The new canonic form is as follows:

$$\dot{w} = \bar{\Lambda} w \tag{3}$$

$$\dot{v} = \Lambda v + h b^T w \tag{4}$$

where

$$\bar{\Lambda} = \begin{pmatrix} -\lambda_1 & 1 & & 0 \\ & \ddots & \ddots & \\ & & \ddots & \ddots \\ 0 & & -\lambda_{n-1} & 1 \\ 0 & & & 0 \end{pmatrix}$$

$$\Lambda = \begin{pmatrix} -\lambda_1 & 1 & & 0 \\ & \ddots & \ddots & \\ & & \ddots & \ddots \\ 0 & & -\lambda_{n-1} & 1 \\ \hdashline -a_1 & \cdots & -a_{n-1} & -(\lambda_n + a_n) \end{pmatrix}$$

where $b^T = (b_1, \ldots, b_n)$; $h^T = (0, \ldots, 0, 1)$; $v_n = y =$ system output; and $w_n = u =$ system input.

Consider Eq. (4); treat w as a system input. Then the state "observer"[2] equation is given by Eq. (5):

$$\dot{\hat{v}} = \Lambda \hat{v} + h b^T w + k(y - \hat{y}) \tag{5}$$

Next select the "adaptive observer gain"†

$$k^T = (0, \ldots, 0, 1, -a_n) \tag{6}$$

and substituting $(\hat{a}, \hat{b}, \hat{w})$ for (a, b, w) in Eq. (5) we get the identifier Eq. (7).

$$\mathring{\hat{v}} = \Lambda^* \begin{pmatrix} \dot{\hat{v}}_1 \\ \vdots \\ \dot{\hat{v}}_{n-1} \\ y \end{pmatrix} + h[\hat{b}^T \hat{w} - \lambda_n \hat{y}] \tag{7}$$

where

$$\Lambda^* = \Lambda + h h^T \lambda_n$$

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† A minimal stationary (nonadaptive) observer results if we select $k^T = (0, \ldots, 0, 1, -\lambda_n)$.

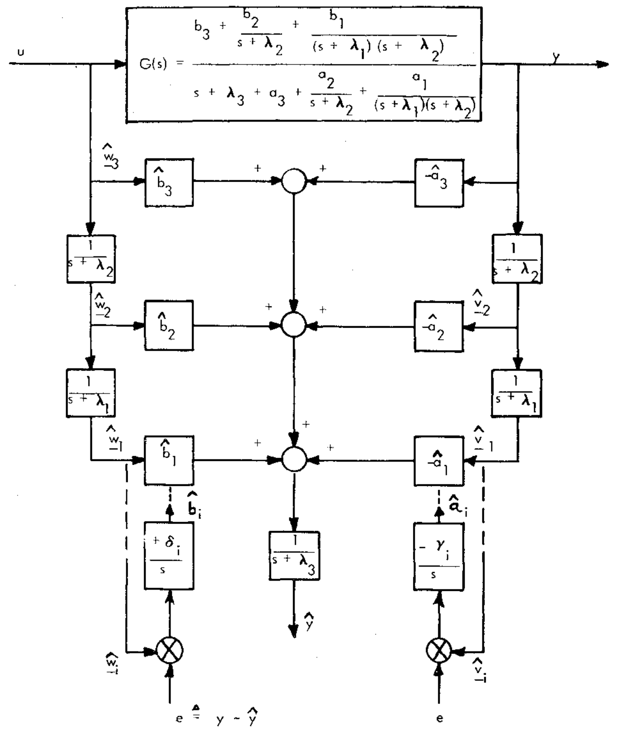

Fig. 1 Parameter and state estimator. The $(\hat{a}, \hat{b})$ are estimates of (a, b). The $(\delta, \gamma, \lambda)$ are design parameters.

The "steepest descent" parameter adjustment law[1] adjusts the parameters towards the $\dot{e} = (\dot{y} - \dot{\hat{y}}) = 0$ surface. From Eqs. (4) and (7) we have:

$$\dot{e} = \sum_{1}^{n-1} -a_i v_i + \sum_{1}^{n-1} \hat{a}_i \hat{v}_i - (a_n + \lambda_n) y + \lambda_n \hat{y} + \hat{a}_n y + b^T w - \hat{b}^T \hat{w} \quad (8)$$

The normal to the $\dot{e} = 0$ hyperplane in the space spanned by the parameter estimates $(\hat{a}, \hat{b})$ is in the direction of "steepest descent." Define $F = 1/2(\dot{e})^2$ as a metric to the $\dot{e} = 0$ surface, then

$$\dot{\hat{a}}_i = -\gamma_i \partial F / \partial \hat{a}_i = -\gamma_i e \hat{v}_i \quad (9)$$

$$\dot{\hat{b}}_i = -\delta_i \partial F / \partial \hat{b}_i = \delta_i e \hat{w}_i \quad (10)$$

This parameter adjustment law completes the adaptive observer identification algorithm, as summarized in Fig. 1. The above parameter adjustment laws can also be developed as a convenient selection satisfying the estimate convergence proof given in the next section.

Proof of Estimate Convergence

The Liapunov proof developed here is based on Ref. 3. This is considered more elegant than the convergence proof given in Ref. 1, since no argument on hyperplane motions is required. Before proceeding with the proof it is convenient to recall the superposition theorem; that is the solution to Eq. (3) in terms of the solution to Eq. (11) is given by Eq. (12).

$$\dot{\hat{w}} = \bar{\Lambda} \hat{w} \quad (11)$$

$$w(t) = \hat{w}(t) + \exp(\bar{\Lambda} t) \Delta w(0) \quad (12)$$

where $\Delta w(0) = w(0) - \hat{w}(0)$. Clearly, if $\lambda_i > 0$, then the influence of any initial estimate error $[\Delta w(0) \neq 0]$ will decay to zero as $t \to \infty$.

Similarly the solution to Eq. (4) in terms of $\hat{v}^T \triangleq (\hat{v}_i, \ldots, \hat{v}_{n-1}, y)$ is:

$$v(t) = \hat{v}(t) + \exp(\bar{\Lambda} t) \Delta v(0) \quad (13)$$

where $v_n = y$, $\Delta v(0) = v(0) - \hat{v}(0)$. Using Eqs. (12) and (13) in Eq. (8) we get

$$\dot{e} = -(a - \hat{a})^T \hat{v} - a^T \exp(\bar{\Lambda} t) \Delta v(0) - \lambda_n e + (b - \hat{b})^T \hat{w} + b^T \exp(\bar{\Lambda} t) \Delta w(0) \quad (14)$$

where

$$a^T = (a_1, \ldots, a_n);$$
$$b^T = (b_1, \ldots, b_n).$$

Now select a Liapunov function

$$V = \tfrac{1}{2} e^2 + \tfrac{1}{2} \sum_{1}^{n} (a_i - \hat{a}_i)^2 / \gamma_i + \tfrac{1}{2} \sum_{1}^{n} (b_i - \hat{b}_i)^2 / \delta_i \quad (15)$$

Using the *stationary system* assumption ($\dot{a} = \dot{b} = 0$), we have:

$$\dot{V} = e\dot{e} + \sum_{1}^{n} [(a_i - \hat{a}_i)(-\dot{\hat{a}}_i)/\gamma_i + (b_i - \hat{b}_i)(-\dot{\hat{b}}_i)/\delta_i] \quad (16)$$

Inserting the "steepest descent" parameter adjustment law, [Eqs. (9) and (10)] we have:

$$\dot{V} = e\dot{e} + e(a - \hat{a})^T \hat{v} - e(b - \hat{b})^T \hat{w} \quad (17)$$

The result of substituting Eq. (14) into Eq. (17) is

$$\dot{V} = -\lambda_n e^2 + e[-a^T \exp(\bar{\Lambda} t) \Delta v(0) + b^T \exp(\bar{\Lambda} t) \Delta w(0)] \quad (18)$$

From the autonomous solution to Eqs. (3) and (4) we have

$$(\dot{y}_A + \lambda_n y_A) = -a^T \exp(\bar{\Lambda} t) v(0) + b^T \exp(\bar{\Lambda} t) w(0) \quad (19)$$

Clearly $(\dot{y}_A + \lambda_n y_A) \to 0$ as $t \to \infty$ if the system to be identified is *stable*; i.e., strictly LHP poles. Hence for a stable system Eq. (18) becomes negative definite; therefore parameter and state estimate convergence is guaranteed.

Implicit in the preceding is the requirement that the input u contain at least n-distinct frequencies (real or complex) such that states (w, v) are excited.[1,3,5,6]

The Nonstationary Systems

It is instructive to consider the nonstationary state and parameter estimation convergence. These considerations aid in initial design parameter selection. Consider the time derivative of Eq. (15) with $\dot{a} \neq 0$ and $\dot{b} \neq 0$. Carrying out steps analogous to those resulting in Eq. (18) results in Eq. (20).

$$\dot{V} = -\lambda_n e^2 + \sum_{1}^{n} [(a_i - \hat{a}_i)\dot{a}_i/\gamma_i + (b_i - \hat{b}_i)\dot{b}_i/\delta_i] + e[-a^T \exp(\bar{\Lambda} t) \Delta v(0) + b^T \exp(\bar{\Lambda} t) \Delta w(0)] \quad (20)$$

Equation (20) indicates that maximum $(\lambda, \delta, \gamma)$ are desirable for maximum state and parameter estimate convergence rate. In practice, upper bounds on the design parameters result from numerical considerations.

Conclusion

It has been shown that for any general input u, the adaptive observer described by Eqs. (7) and (9–11) will asymptotically yield the states and parameters of an nth-order linear time-invariant system, Eq. (2). This adaptive observer does not require auxiliary signals to be fed back to it; hence its implementation is very simple (see Fig. 1).

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